Student name: ______________________

2021 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Sydney Technical High School

Mathematics Advanced

23 August 2021

General Instructions

- Reading time – 10 minutes
- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In questions 11-39, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 1-7)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8-36)

- Attempt questions 11-39
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1. What is the gradient of the line $3x + 4y + 5 = 0$

A. $-\frac{3}{4}$

B. $\frac{3}{4}$

C. $-\frac{4}{3}$

D. $\frac{4}{3}$

2. Estragon has scored $90\%, 78\%, 81\%,$ and 83% on his first four class tests. After the fifth class test, his mean mark is increased by 1%.

What was Estragon's mark in his fifth class test ?

A. 92%

B. 84%

C. 88%

D. 83%

3. Which of the following is equal to $\dfrac{\log_4 16}{\log_4 2}$?

A. 8

B. $\log_4 8$

C. $\log_4 12$

D. 4

4. The function $f(x) = \dfrac{1}{x}$ is translated 3 units up and 2 units right to produce $y = g(x)$.

Which of the following is the equation of the translated function $g(x)$?

A. $g(x) = \dfrac{1}{x+2} - 3$

B. $g(x) = \dfrac{1}{x+2} + 3$

C. $g(x) = \dfrac{1}{x-2} + 3$

D. $g(x) = \dfrac{1}{x-2} - 3$

5. Given that $\tan\theta = \frac{3}{2}$ for $0 < \theta < \pi$, what is the exact value of $\sin\theta$?

A. $-\frac{2}{\sqrt{13}}$

B. $\frac{3}{\sqrt{13}}$

C. $-\frac{3}{\sqrt{13}}$

D. $\frac{2}{\sqrt{13}}$

6. What is the domain and range of $f(x) = \sqrt{x-4}$?

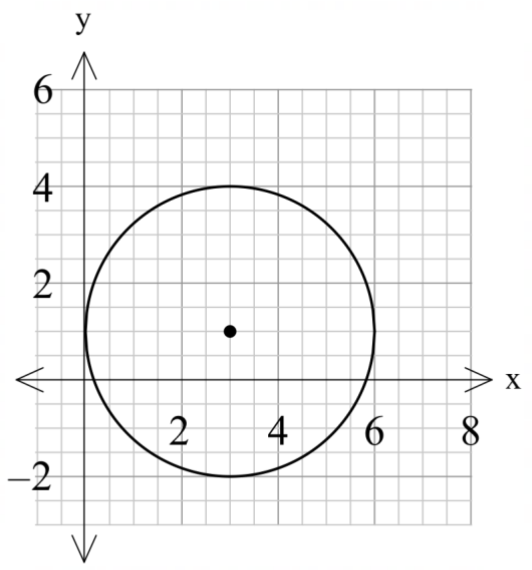
A. Domain: $(-\infty, \infty)$ and Range: $(0, \infty)$

B. Domain: $[4, \infty)$ and Range: $[0, \infty)$

C. Domain: $[0, \infty)$ and Range: $[4, \infty)$

D. Domain: $[2, \infty)$and Range: $[0, \infty)$

7.



Which of the following is the equation for the circle shown in the diagram above?

A. $x^2 + 6x + y^2 + 2y + 1 = 0$

B. $x^2 - 6x + y^2 + 2y - 1 = 0$

C. $x^2 - 6x + y^2 - 2y + 1 = 0$

D. $x^2 - 6x + y^2 - 2y - 1 = 0$

8. For events A and B over a sample space $P(A \cap B) = 0.2$ and $P(A|B) = 0.25$

What is $P(B)$?

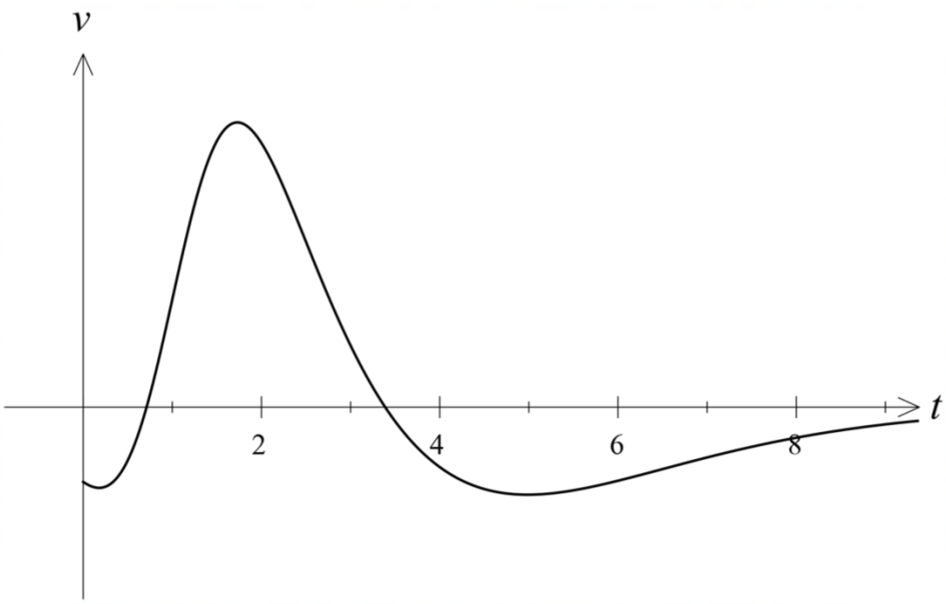
A. 0.45

B. 0.05

C. 0.75

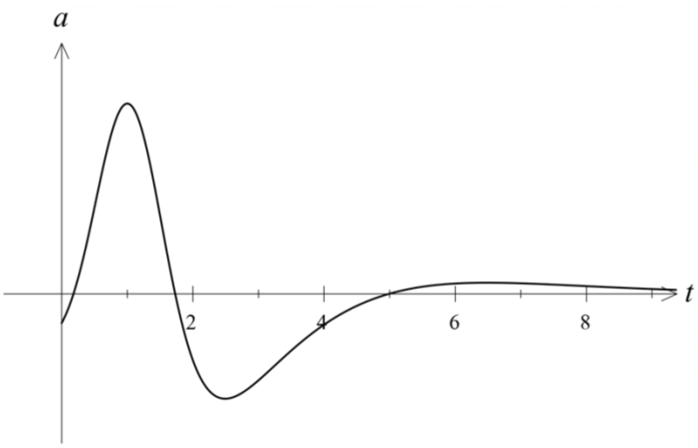
D. 0.8

9. The graph below is velocity/time graph for a particle



Which of the following could be the acceleration graph ?

A.



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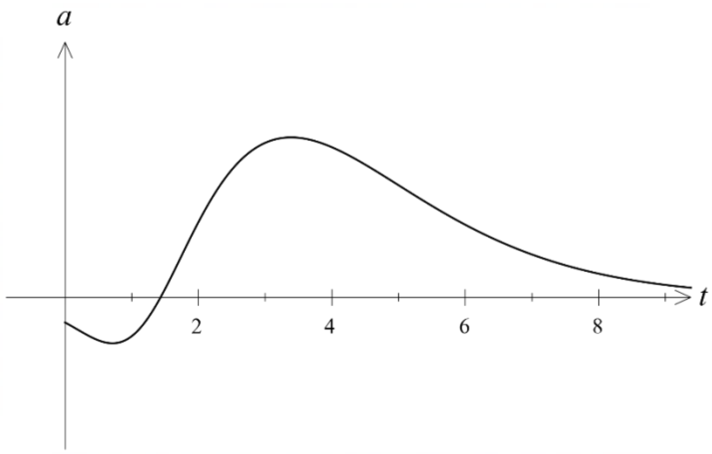
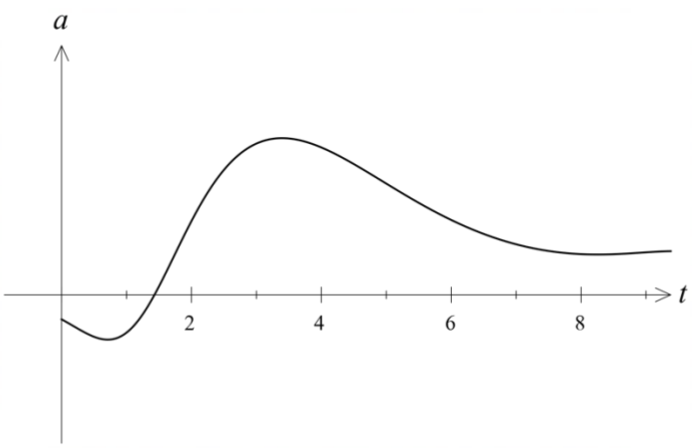
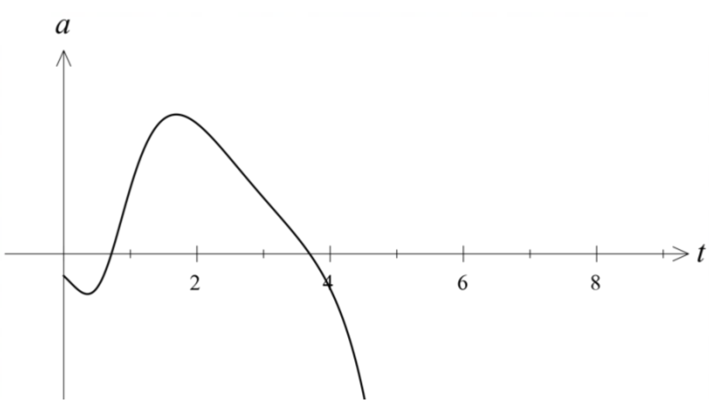


C.



D.



10. Which of the following is the gradient of the normal to $y = \log_2 x$ at the point $(8,3)$?

A. $-\dfrac{1}{8\ln 2}$

B. $-8\ln 2$

C. $\dfrac{1}{8\ln 2}$

D. $8\ln 2$

Please turn over for Multiple Choice answer sheet

Mathematics Advanced
Section II question and answer booklet

90 marks

Attempt Questions 11-39
Allow about 2 hours and 45 minutes for this section

Part 1: Attempt Questions 11-29 (44) marks
Part 2: Attempt Questions 30-39 (46) marks

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 37−41 of Booklet. If you use this space, clearly indicate which question you are answering as well as making a notation on the original question

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Please turn over

Section II Part 1: Attempt Questions 11-29 (44) marks

- Extra writing space is provided on pages 37−41 of Booklet. If you use this space, clearly indicate which question you are answering as well as making a notation on the original question

Question 11 (1 mark)

Differentiate $3 + \tan 8x$ 1

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Question 12 (2 marks)

Fully simplify $\dfrac{x^2 - 4}{2x^2 + 3x - 2}$ 2

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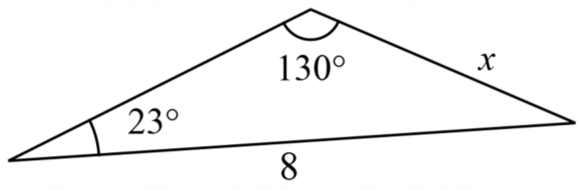
Please turn over for question 13

Question 13 (1 marks)

Find the value of x correct to 1 decimal place 1



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Question 14 (2 marks)

Evaluate $\int_0^3 (6x^2 + 2x + 5)\,dx$ 2

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Question 15 (4 marks)

Differentiate and fully simplify where possible

(a) $10^{(x+4)}$ **1**

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(b) e^{x^2-1} **1**

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(c) $\ln(\cos x)$ **2**

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Question 16 (2 marks)

find $\int (1 + \tan^2 x)\ dx$ 2

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Question 17 (1 mark)

The function $f(x)$ is a probability density function. 1

$$f(x) = \begin{cases} 0.2 & \text{for} \quad 0 \le x < k \\ 0 & \text{otherwise} \end{cases}$$

Find the value of k

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Question 18 (3 marks)

AstraZeneca and Pfizer are currently the two Covid-19 Vaccines available in Australia. 50 unvaccinated Newtown residents were asked about their willingness to be vaccinated by either vaccine over the next 3 months.

- 26 residents said they were willing to be vaccinated by the Pfizer vaccine
- 22 residents said they were willing to be vaccinated by the AstraZeneca vaccine
- 12 residents said they were NOT willing to be vaccinated

(a) A random resident from the group was selected. By using a Venn diagram or otherwise, find the probability that they were willing to be vaccinated by either vaccine **2**

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(b) Two residents were selected (without replacement). What is the probability that at least one of them was willing to be vaccinated ? **1**

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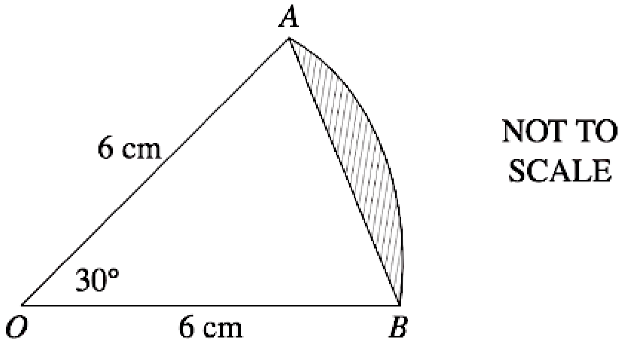
Question 19 (2 marks)

In the diagram, OAB, is a sector of a circle with centre O and a radius of 6 cm, where $\angle AOB = 30°$



(a) Find the exact value of the area of triangle OAB **1**

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(b) Find the exact area of the shaded segment **1**

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Question 20 (2 marks)

Solve $|4x - 2| = 10$ 2

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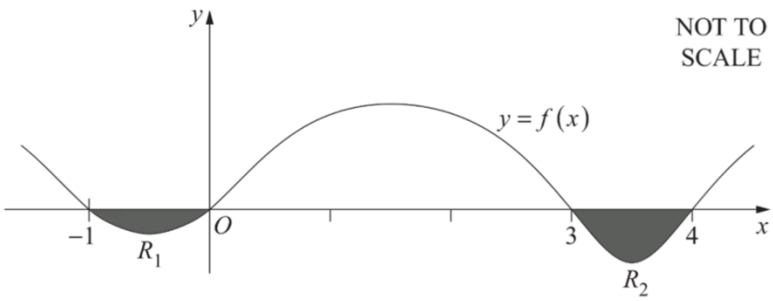
Question 21 (2 marks)

The diagram below shows the graph of $y = f(x)$ with intercepts at $x = -1, 0, 3, \text{and } 4$ 2



- The area of shaded region R_1 is 4
- The area of shaded region R_2 is 5
- It is given that $\int_0^4 f(x)\, dx = 15$

What is the value of $\int_{-1}^{3} f(x)\, dx$?

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Question 22 (4 marks)

Atticus owns a biased six-sided die which has the following discrete probability distribution

x	1	2	3	4	5	6
P(X = x)	0.25	0.25	0.2	0.1	0.1	0.1

(a) Find $P(3 < X \leq 6)$ **1**

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(b) Show that the expected value $E(X) = 2.85$ **1**

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(c) At a school fund raiser, Atticus uses his biased die to run a game, where students pay \$3.50 per game to roll the biased die and in turn win \$1 multiplied by the number that they have rolled **2**

How much profit or loss would Atticus expect to make for 200 games?

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Question 23 (2 marks)

The 2021 Semester one report marks for a Year 12 Advanced Mathematics cohort at a particular selective boys high school is normally distributed with a mean of 76 and a standard deviation of 11

(a) What is the probability that a randomly selected student from the cohort will have a mark between 54 and 98 ? **1**

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(b) A student is selected randomly from the cohort. What is the probability that his mark is less than or equal to 65? **1**

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Question 24 (2 marks)

Prove that $(\sin x + \cos x)(\sec x - \operatorname{cosec} x) = \tan x - \cot x$ **2**

Question 25 (2 marks)

Find $\int x(x^2+5)^9 \, dx$ **2**

Question 26 (3 marks)

Five values of a function $f(x)$ are shown in in the table below.

x	20	25	30	35	40
$f(x)$	10	12	25	28	20

Use the trapezoidal rule with the five values given in the table to estimate 3

$$\int_{20}^{40} f(x)\,dx$$

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Question 27 (3 marks)

(a) Differentiate $y = xe^{2x}$ 1

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(b) Hence find the <u>exact</u> value of $\int_{0}^{2} e^{2x}(3+6x)\,dx$ 2

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Question 28 (3 marks)

The following table shows the number of absences from class versus the common test mark in a mathematics class of 11 students (student Joseph K missed the test and so does not appear in the table)

Student	A	B	C	D	E	F	G	H	I	J
Number of absences	0	1	1	2	3	3	6	8	11	16
Mark %	90	85	88	84	82	80	75	60	55	34

(a) Find the equation of the least-squares line of best fit in terms of number of absences (n) and the mark (m) (correct to 3 decimal places) **2**

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(b) Student Josef K who missed the test, was absent from 13 maths lessons in total. **1**
His teacher estimated his mark using the equation of the line of best fit.

What was Josef's estimated mark? (correct to the nearest whole number)

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Question 29 starts over the page

Question 29 (3 marks)

The temperature of a freshly served bowl of phở bò from Tan Viet Noodle House is given by the following equation:

$$T = 22 + 60e^{-0.1t}$$

Where T is the temperature in degrees Celsius

And t is the time in minutes.

(a) To the nearest degree, what is the temperature of the phở after 1 minute ? **1**

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(b) How long will it take for the temperature of the phở to drop to $62°$ **2**

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End of section II part 1

Please turn over

Section II Part 2: Attempt Questions 30-39 (46) marks

Question 30 (5 marks)

Use the following standard Z-table for this question.

	first decimal place									
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1.	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2.	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3.	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

(a) What is $P(X \leq 1.8)$? **1**

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(b) What is $P(-1.2 \leq X \leq 0.3)$? **2**

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(c) The birthweight of babies is known to be normally distributed. According to data which covers Australian pregnancies between 1998 and 2007, the mean (μ) birthweight for boys was 3632 grams with a standard deviation (σ) of 430 grams.

What is the probability that a randomly selected newborn boy will weigh less than 3890 grams ? **2**

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Question 31 (7 marks)

Consider the function $f(x) = x^3 + 3x^2 - 9x$

(a) Show that $f(x)$ has 2 stationary points at $P\ (1, -5)$ and $Q(-3,27)$ and determine their nature **3**

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(b) Find any points of inflexion **2**

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Question 31 continues over next page

Question 31 (continued)

(c) Sketch the graph marking and labelling any stationary points or points of inflexion. **2**
Do **NOT** determine the x intercepts of the curve

Question 32 (3 marks)

A continuous random variable X has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{x}{2} + \frac{1}{4} & \text{for} \quad 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the cumulative distribution function **2**

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(b) Find $P(1.6 \leq X \leq 2)$ **1**

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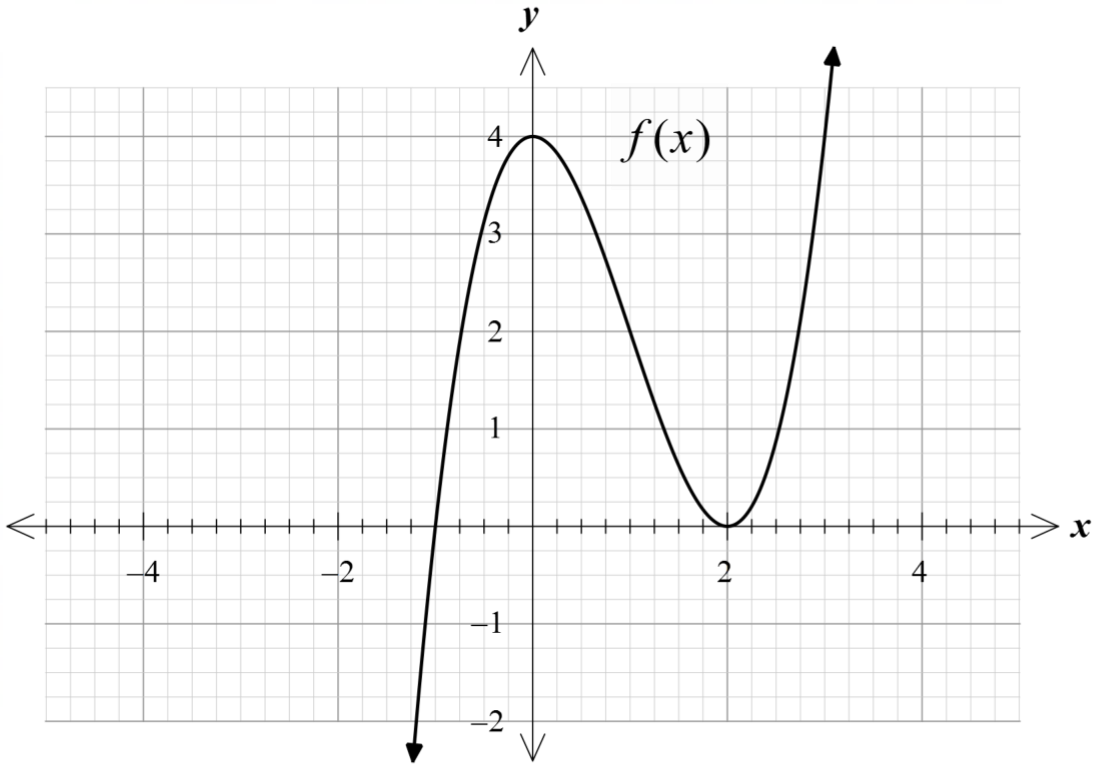
Question 33 (3 marks)

The graph of $f(x) = x^3 - 3x^2 + 4$ is shown below



(a) On the diagram above, make an accurate sketch of $g(x) = -x^2 + x + 2$ marking the vertex and all intercepts **2**

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(b) Hence, using the graphical method, find the number of solutions to the following equation

$$x^3 - 3x^2 + 4 \;=\; -x^2 + x + 2$$ **1**

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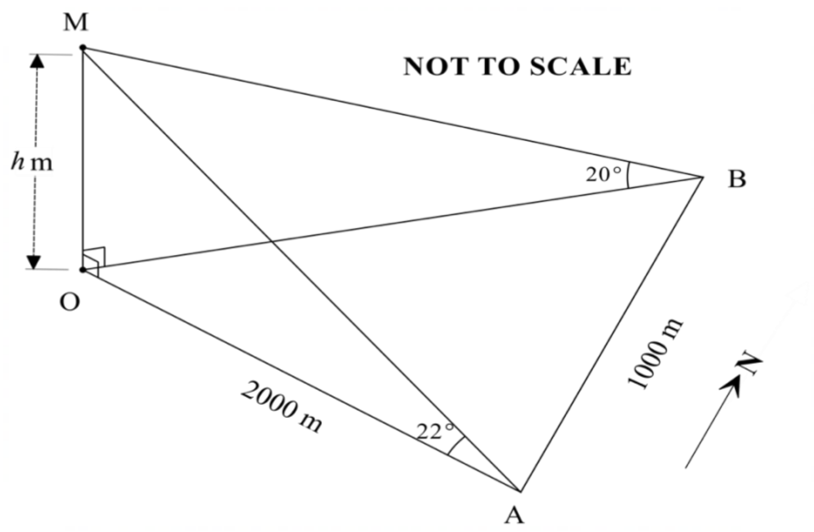
Question 34 (5 marks)

Sancho Panza rides a donkey 1000 metres due north along a straight level road from point A to point B. A is 2000 metres from point O, the base of the vertical mountain OM, where M is the top of the mountain. O is on the same horizontal plane as the road and directly below M. The mountain is to the left of the road and its height above point O is h metres

- From point A, the angle of elevation to the top of the mountain is 22°
- From point B, the angle of elevation to the top of the mountain is 20°



(a) Show that $h = 808$ metres to the nearest metre **1**

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(b) Find OB to the nearest metre **1**

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Question 34 continues over next page

Question 34 (continued)

(c) Hence, find the bearing of O from B (nearest degree) 3

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Turn over for Question 35

Question 35 (5 marks)

A tank initially holds 8100 litres of Water. The water drains from the bottom of the tank. A mathematical model predicts that the volume, V litres, of water that will remain in the tank after t minutes is given by

$$V = 8100\left(1 - \frac{t}{90}\right)^2$$

(a) How long does the tank take to drain fully? **1**

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(b) What volume does the model predict will remain after 20 minutes ? **1**

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(c) At what rate does the model predict that the water will be draining from the tank after 30 minutes ? **2**

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Question 35 continues over next page

Question 35 (continued)

(d) At what time does the model predict that the water will drain from the tank at its fastest rate ? **1**

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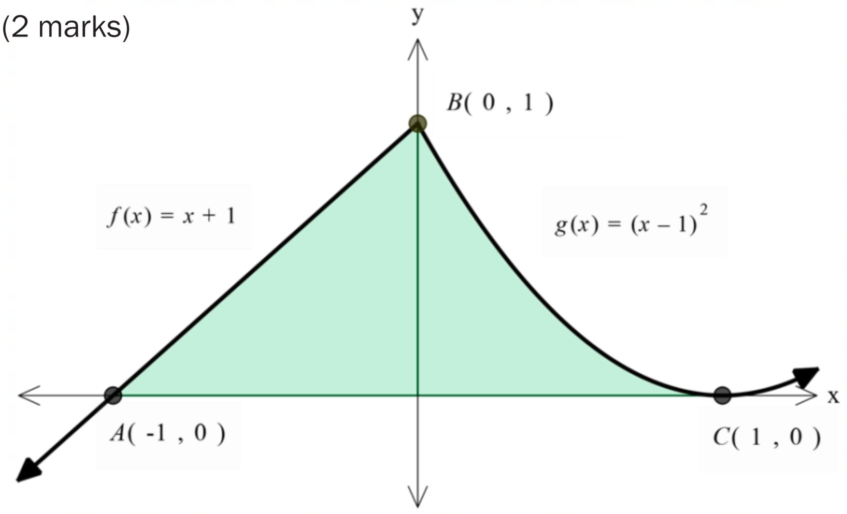
Turn over for Question 36

Question 36 (2 marks)



The diagram above shows portions of the graphs for $f(x) = x + 1$ and $g(x) = (x - 1)^2$

- $f(x)$ and $g(x)$ intersect at the point $B\ (0, 1)$
- The $x -$ intercept of $f(x)$ is $A\ (-1, 0)$
- $g(x)$ touches the $x -$ axis at point $C\ (1, 0)$

Find the exact shaded area that is enclosed by the $f(x)$, $g(x)$, and the $x -$ axis **2**

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Question 37 (6 marks)

A particle moves in a straight line. Its velocity v m/s at time t seconds is given by

$$v = 2 - \frac{4}{t+1}$$

(a) Show that the particle is at rest when $t = 1$ **1**

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(b) The particle's initial displacement is 1 m. Find its displacement when at rest (3 decimal points) **3**

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Question 37 continues over next page

Question 37 (continued)

(c) Find the expression for the accelaration of the particle at time t **1**

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(d) Explain why the displacement of the particle is always positive **1**

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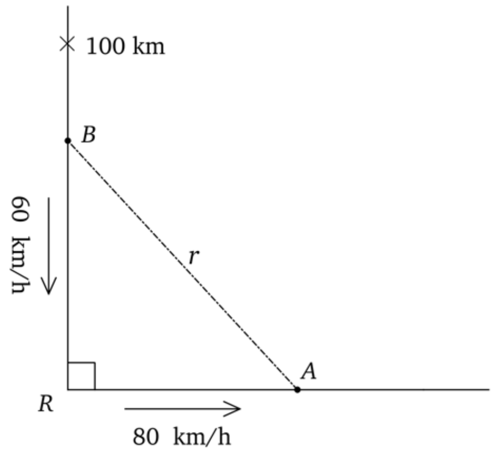
Question 38 (5 marks)

Two straight roads meet at R at an angle of $90°$. At time $t = 0$ car A leaves R on one road, and car B is 100 km from R on the other road. Car A travels away from R at a speed of 80 km/h, and car B travels towards R at a speed of 60 km/h.



The distance between the cars at time t hours is r km.

(a) Show that $r^2 = 2000(5t^2 - 6t + 5)$ **2**

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Question 38 continues over next page

Question 38 (continued)

(b) Show that $\frac{d(r^2)}{dt} = 20000t - 12000$ **1**

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(c) Hence or otherwise find the minimum distance (r) between the cars **2**
(justify your answer with relevant mathematical arguments)

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Question 39 (5 marks)

Between 8 am and 8 pm on July 18, 2021, the height of the tide in Sydney harbour was given by

$$h(t) = 1 + 0.4\sin\left(\frac{\pi}{6}t\right) \text{ for } 0 \leq \text{t} < 12$$

Where h is the height of the tide in metres, and t is in hours with $t = 0$ at 8 am

(a) What is the period of h(t) **1**

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(b) What was the height $h(t)$ at low tide, and at what time did this low tide occur **2**

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(c) A cruise ship is only able to enter the harbour if the tide is at least 1.2 m. Find all the times between 8 am and 8 pm on July 18 2021 that the cruise ship was able to enter the harbour **2**

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End of paper

Solutions 2021 STHS
Advanced Trial

Section 1 (MC)

1	2	3	4	5	6	7	8	9	10.
A	C	D	C	B	B	C	D	A	B

1. $3x+4y+5=y$

$y = \dfrac{-3x-5}{4}$ $\quad\therefore$ gradient $= -\dfrac{3}{4}$ $\therefore$ (A)

2. Original average $= \dfrac{90\% + 78\% + 81\% + 83\%}{4} = \dfrac{332}{4} = 83\%$

new mean $= 84\%$

$\dfrac{x+332}{5} = 84$

$\therefore x = 5 \times 84 - 332 = 88\%$ $\quad\therefore$ (C)

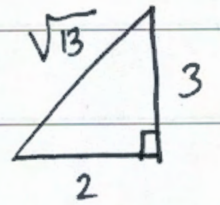
3. $\dfrac{\log_4 16}{\log_4 2} = \dfrac{\log_4 2^4}{\log_4 2} = \dfrac{4 \log_4 2}{\log_4 2} = 4$ $\quad\therefore$ (D)

4. $\dfrac{1}{x-2} + 3$ $\quad\therefore$ (C)

($x-2$: 2 right; $+3$: 3 up)

P02

5.



$\tan\theta = \frac{3}{2}$ $\quad hyp = \sqrt{3^2+2^2}$
$= \sqrt{13}$

tan +ve ∴ in Q1 or Q3
$0 < \theta < \pi$
∴ in Q1

$\sin\theta = \frac{opp}{hyp} = \frac{3}{\sqrt{13}}$ $\quad$ sin +ve as in Q1

∴ (B)

6. $f(x) = \sqrt{x-4}$ $\quad$ x must be ≥ 4

y must be ≥ 0

∴ D: $[4, \infty)$ R: $[0, \infty)$ $\quad$ ∴ (B)

7. Radius $= 3$
centre $(3, 1)$

∴ equation ⇒ $(x-3)^2 + (y-1)^2 = 3^2$

$x^2 - 6x + 9 + y^2 - 2y + 1 = 9.$

$x^2 - 6x + y^2 - 2y + 1 = 0$ $\quad$ ∴ (C)

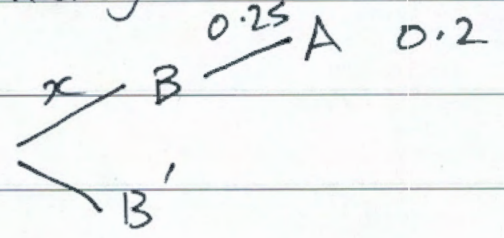
8. $P(A|B) = \frac{P(A \cap B)}{P(B)}$

∴ $P(B) = \frac{P(A \cap B)}{P(A|B)} = \frac{0.2}{0.25}$

$= 0.8$

8 continued...

alternatley

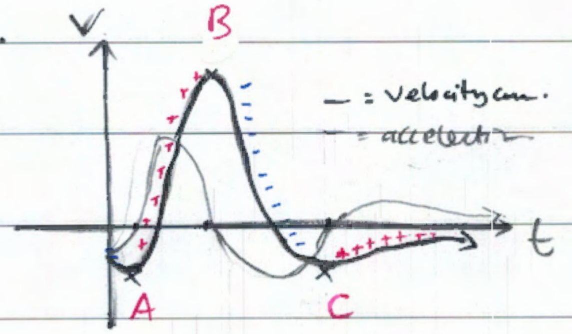


$\therefore 0.25x = 0.2$

$x = \frac{0.2}{0.25}$

$= 0.8$ $\therefore$ (D)

9.



The velocity curve has 3 turning points $\therefore$ acceleration curve must have exactly 3 x-intercepts so we eliminate B, C, D $\therefore$ Must be (A)

10. $f(x) = \log_2 x$ $f'(x) = \frac{1}{\ln(2)x}$

@ (8, 3)

$m_{tan} = f'(x) = \frac{1}{\ln(2) \times 8}$

$m_{normal} = \frac{-1}{m_{tan}} = -8\ln(2)$ $\therefore$ (B)

Solutions to 2021
STHS MATHS
ADVANCED TRIAL

SECTION 2

11) $y = 3 + \tan 8x$

$$\frac{dy}{dx} = 8\sec^2 8x$$

12) $$\frac{x^2-4}{2x^2+3x-2} = \frac{(x+2)(x-2)}{(2x-1)(x+2)}$$

$$= \frac{x-2}{2x-1}$$

13) $$\frac{x}{\sin 23^\circ} = \frac{8}{\sin 130^\circ}$$

$$x = \frac{8\sin 23^\circ}{\sin 130^\circ}$$

$= 4.1$ to 1 d.p.

P05

14) $\int_0^3 (6x^2+2x+5)\,dx = \left[\frac{6x^3}{3} + \frac{2x^2}{2} + 5x\right]_0^3$

$= (2\times3^3 + 3^2 + 5\times3) - (0)$

$= 78$

15)

(a) $\frac{d}{dx} 10^{(x+4)} = 1 \times \log_e(10) \times 10^{(x+4)}$

$= \log_e(10)\, 10^{(x+4)}$

(b) $\frac{d}{dx} e^{x^2-1} = 2x e^{x^2-1}$

(c) $\frac{d}{dx} \ln(\cos x) = \frac{\frac{d}{dx}\cos x}{\cos x}$

$= \frac{-\sin x}{\cos x}$

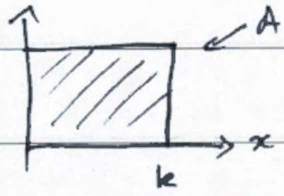
$= -\tan x$

16) $\int (1+\tan^2 x)\,dx = \int \sec^2 x\,dx$ (Pythagorean identity)

$= \tan x + C$

Q17) $f(x) = \begin{cases} 0.2 & \text{for } 0 \le x \le k \\ 0 & \text{otherwise.} \end{cases}$

PDF $\therefore$ area $= 1$



$0.2k = 1$.

$\therefore k = 5$

alternate solution

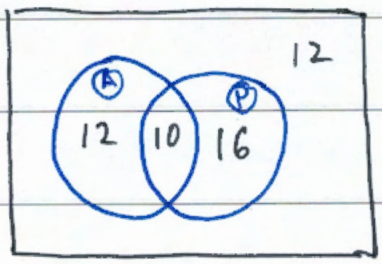
$$\int_0^k 0.2\,dx = 1$$

$$[0.2x]_0^k = 1$$

$$0.2k = 1$$

$$\therefore k = 5$$

Q18)



both $= (22+26+12) - 50$

$= 10$

(a) $P(\text{willing either vaccine}) = P(\text{both}) = \frac{10}{50} = \frac{1}{5}$

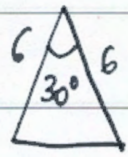
(b) $P(\text{at least one willing}) = 1 - P(\text{neither willing})$

$$= 1 - \frac{12}{50} \times \frac{11}{49}$$

$$= \frac{1159}{1225} \quad (0.946122449)$$

P07

Q19) (a)



$A = \frac{1}{2} ab \sin\theta$

$= \frac{1}{2} \times 6 \times 6 \times \sin 30^\circ \ cm^2$

$= 9 cm^2$

(b) $A_{segment} = A_{sector} - A_{triangle}$.

$= \pi \times 6^2 \times \frac{30}{360} - 9 cm^2$

$= 3\pi - 9 \ cm^2 \qquad (0.425 cm^2)$

Q20) $|4x - 2| = 10$

$4x - 2 = 10 \qquad 4x - 2 = -10$

$4x = 12 \qquad 4x = -8$

$x = 3 \qquad x = -2$

$\therefore x = 3, -2$

Q21) $\int_0^4 f(x)dx = \int_0^3 f(x)dx + \int_3^4 f(x)dx$

$15 = \int_0^3 f(x)dx - 5 \quad \therefore \int_0^3 f(x)dx = 15 - -5$

$= 20$

$\int_{-1}^3 f(x)dx = \int_{-1}^0 f(x)dx + \int_0^3 f(x)dx. = -4 + 20$

$= 16$

P08

Q22) (a) $P(3 < X \leq 6) = 0.1 + 0.1 + 0.1$

$= 0.3$

(b) $E(X) = 1\times0.25 + 2\times0.25 + 3\times0.2 + 4\times0.1 + 5\times0.1 + 6\times0.1$

$= 2.85$

(c) expected profit per game = (cost per game − \$ × E(X))

$= \$3.50 - \$2.85$

$= \$0.65$

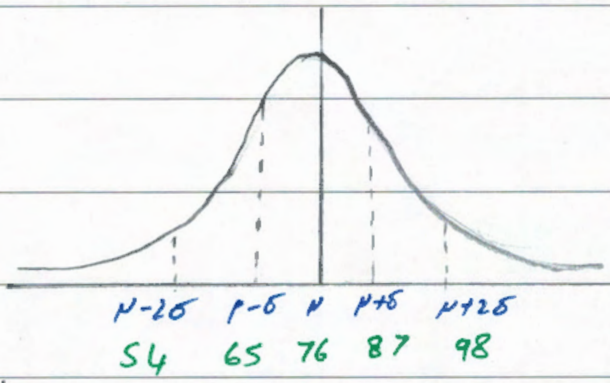
∴ expected profit for 200 games = $200 \times \$0.65$

$= \$130$

Q23)

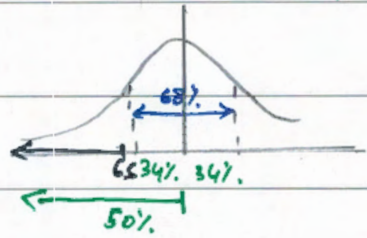
(a) $P(54 \leq X \leq 98) = 95\%$

(imperical rule) 2σ



(b) $P(X \leq 65) = 50\% - 34\%$

$= 16\%$



Q24)

$LHS = (\sin x + \cos x)(\sec x - \csc x)$

$= \sin x \sec x + \cos x \sec x - \sin x \csc x - \cos x \csc x.$

$= \frac{\sin x}{\cos x} + \frac{\cos x}{\cos x} - \frac{\sin x}{\sin x} - \frac{\cos x}{\sin x}$ (=1, =1)

$= \tan x - \cot x$

$= RHS$

Q25) $\int x(x^2+5)^9\,dx$

$= \frac{1}{2}\int 2x(x^2+5)^9\,dx$

$= \frac{1}{2}\int f'(x)[f(x)]^n\,dx$ where $f(x) = (x^2+5)$, $n = 9$

$= \frac{1}{2} \times \frac{1}{n+1}[f(x)]^{n+1} + C$

$= \frac{1}{2} \times \frac{1}{10} \times (x^2+5)^{10} + C$

$= \frac{(x^2+5)^{10}}{20} + C$

Q26) trapezoidal rule.

$n = 4$ (# of subintervals not # function values)

$\int_{20}^{40} f(x)\,dx \simeq \frac{1}{2} \times \text{length of subinterval} \times \{\text{first } y + \text{last } y + 2 \times \text{intermediate values}\}$

$= \frac{1}{2} \times 5 \times \{10 + 20 + 2 \times (12 + 25 + 28)\}$

$= 400$

Q27) (a) $y = xe^{2x}$

$u = x \quad du = 1$
$v = e^{2x} \quad dv = 2e^{2x}$

$\frac{dy}{dx} = vdu + udv$

$= 1 \times e^{2x} + x \times 2e^{2x}$

$= (1+2x)e^{2x}$ or $e^{2x} + 2xe^{2x}$

(b) $\int_0^2 e^{2x}(3+6x) = 3\int_0^2 e^{2x}(1+2x)\,dx$

$= 3 \times \left[xe^{2x}\right]_0^2$ (from (a) reverse chain rule)

$= 3 \times (2e^{2\times 2} - 0)$

$= 6e^4$

Q28) (a) use calculator for least squares regression line

$A = 90.920$

$B = -3.455$ where $y = A + Bx$.

$\therefore \; m = -3.455n + 90.920$ where n = number of absences
m = mark.

(b) $m = -3.455 \times 13 + 90.920$

$= 46$ (to nearest whole number)

Q29)

(a) $T = 22 + 60e^{-0.1\times1}$

$= 76.29..°$

$= 76°$ to nearest degree

(b) $62 = 22 + 60e^{-0.1t}$

$60e^{-0.1t} = 40$

$e^{-0.1t} = \frac{2}{3}$

$-0.1t = \ln\frac{2}{3}$

$t = 4.05$ min (or 4 minutes 3 seconds)

END SECTION 2 PART 1.

section 2 part 2

Q30) (a)

$P(X \leq 1.8) = 0.9641$ (from Z table)

(b) $P(-1.2 \leq X \leq 0.3) = P(X \leq 0.3) - P(X \leq -1.2)$

$= P(X \leq 0.3) - (P\, X \geq 1.2)$ (Symmetry)

$= P(X \leq 0.3) - (1 - P(X \leq 1.2))$

$= 0.6179 - 1 + 0.8844$

$= 0.5028$

(c) $Z = \frac{3890 - 3632}{430} = 0.6$

$P(X \leq 0.6) = 0.7257$ (from Z table)

Q31)
(a) $f(x) = x^3 + 3x^2 - 9x$

$f'(x) = 3x^2 + 6x - 9$

$= 3(x^2 + 2x - 3)$

$= 3(x+3)(x-1)$

$f'(x) = 0$

$3(x+3)(x-1) = 0$

$x = -3$ or 1

$f(-3) = (-3)^3 + 3\times(-3)^2 - 9\times 3$

$= 27$

$f(1) = 1^3 + 3\times 1^2 - 9\times 1$

$= -5$

∴ stationary points are

$(-3, 27)$, $(1, -5)$ as required

Nature

x	-4	-3	0	1	2
$f'(x)$	+ve	0	−ve	0	+ve
	/	⌒	\	⌣	/

∴ $(-3, 27)$ is a local maxim.
$(1, -5)$ is a local minim.

(b) $f''(x) = 6x + 6$

$f''(x) = 0$

$6x + 6 = 0$ ∴ $x = -1$

$y = (-1)^3 + 3(-1)^2 - 9\times -1$

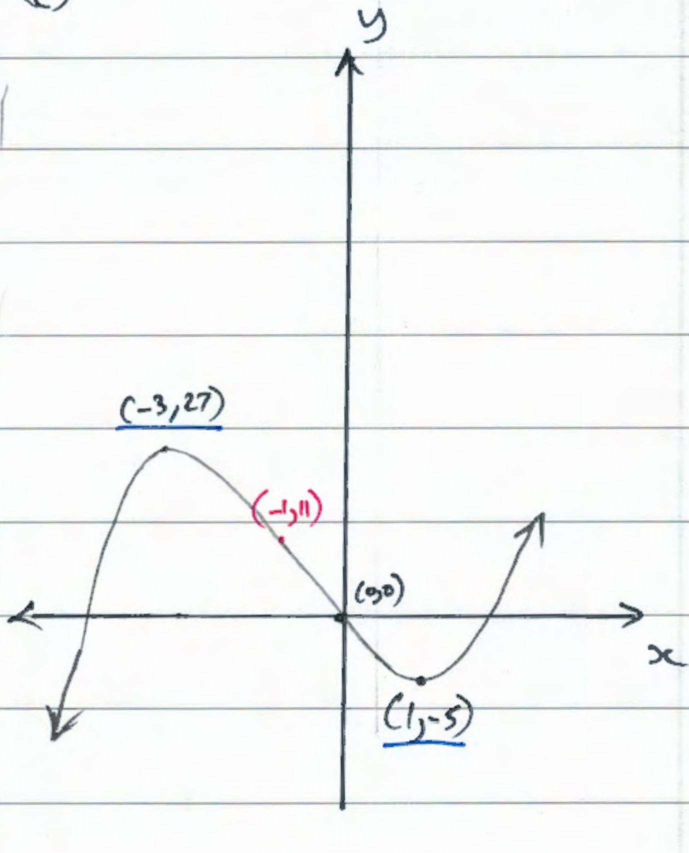
$= 11$

$(-1, 11)$ is a point of inflexion

$f''(-1) = 0$

$f'(-1) \neq 0$ (from part a)

(c)



(PB)

Q32)

$$f(x) = \begin{cases} \frac{x}{2} + \frac{1}{4} & 1 \le x \le 2. \\ 0 & \text{elsewhere} \end{cases}$$

$$F(x) = \int_1^x f(x)\,dx$$

$$= \begin{cases} \int_1^x (\frac{x}{2} + \frac{1}{4})\,dx & 1 \le x \le 2. \\ 0 & \text{elsewhere.} \end{cases}$$

$$= \begin{cases} \left[\frac{x^2}{4} + \frac{x}{4}\right]_1^x & 1 \le x \le 2. \\ 0 & \text{elsewhere} \end{cases}$$

$$F(x) = \begin{cases} \left(\frac{x^2}{4} + \frac{x}{4}\right) - \left(\frac{1^2}{4} + \frac{1}{4}\right) & 1 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$F(x) = \begin{cases} \frac{x^2 + x - 2}{4} & 1 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases} \quad \left(\text{or } \frac{x^2}{4} + \frac{x}{4} - \frac{1}{2}\right)$$

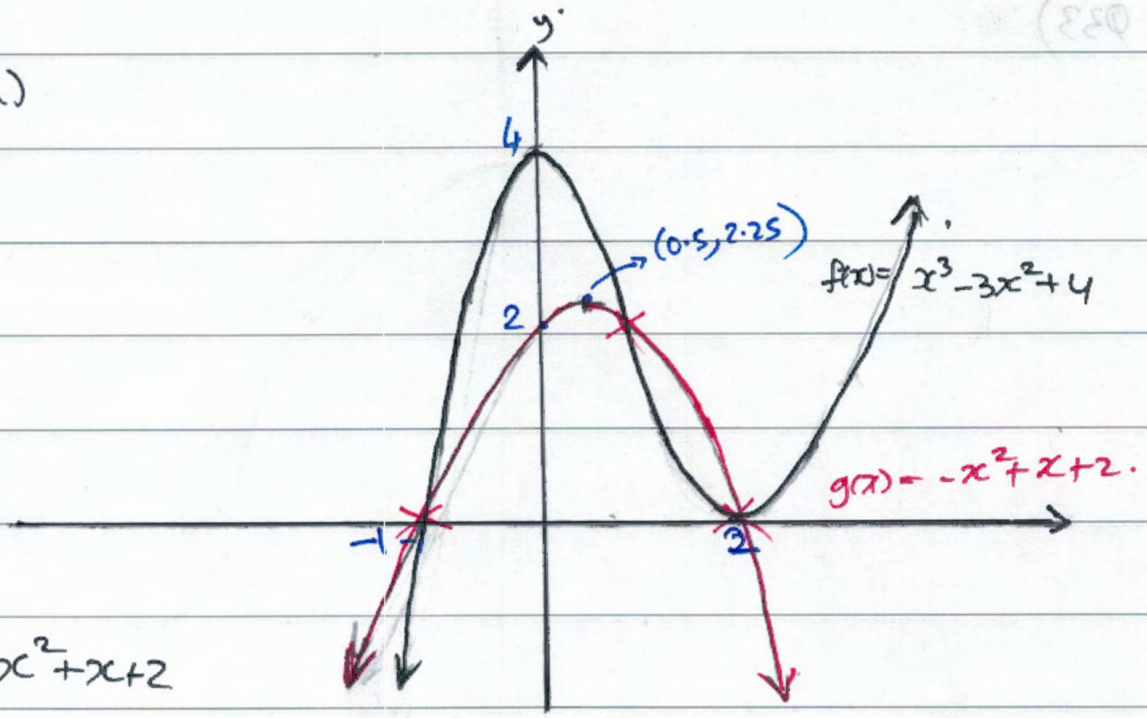
(b) $P(1.6 \le x \le 2) = F(2) - F(1.6)$

$$= \left(\frac{2^2}{4} + \frac{2}{4} - \frac{1}{2}\right) - \left(\frac{1.6^2}{4} + \frac{1.6}{4} - \frac{1}{2}\right)$$

$$= 0.46 \quad \left(\text{or } \frac{23}{50}\right)$$

Q33)

(a)



$g(x) = -x^2 + x + 2$

$= -(x^2 - x - 2)$

$= -(x-2)(x+1)$

x-intercepts = 2, -1   y intercept = 2

Vertex $x = 0.5$

$y = 2.25$

vertex = (0.5 , 2.25)

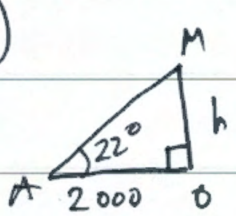
(b) Solutions to $x^2 - 3x^2 + 4 = -x^2 + x + 2$ are

the intersections of f(x) and g(x).

∴ # solutions = 3

Q34)

(a)



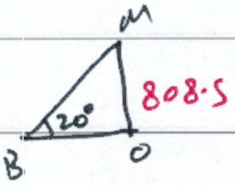
$\frac{OM}{OA} = \tan 22°$ (h / 2000)

$\therefore h = 2000 \tan 22°$

$= 808.5$ m

$= 808$ m to nearest metre as required.

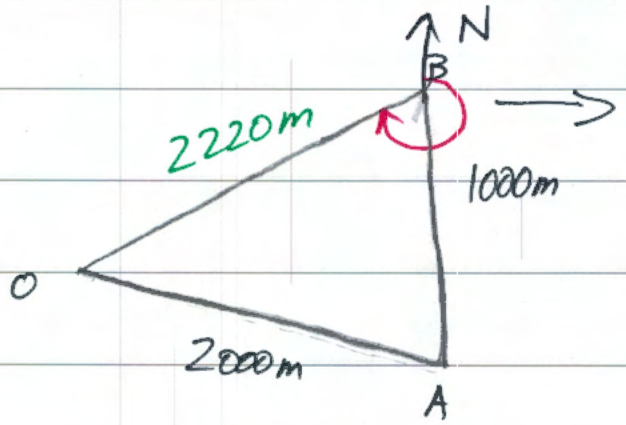
(b) $\frac{OM}{OB} = \tan 20°$



$\therefore OB = \frac{h}{\tan 20°} = 2220.105$ m

$= 2220$ m to nearest metre

(c)



bearing of O from B $= 180° + \angle B$

$\cos B = \frac{2220^2 + 1000^2 - 2000^2}{2 \times 1000 \times 2200} = 0.483$

$B = \cos^{-1} 0.483 = 63.994.$

$\therefore$ bearing of O from B $= 180 + 64°$

$= 244°T$

Q35) (a) $V = 8100\left(1 - \frac{t}{90}\right)^2$

to drain fully $V = 0$ $\therefore \left(1 - \frac{t}{90}\right)^2 = 0$

$\therefore t = 90$ minutes

(b) $V = 8100\left(1 - \frac{20}{90}\right)^2$ L

$= 4900$ L

(c) $\frac{dV}{dt} = -\frac{1}{90} \times 8100 \times 2 \times \left(1 - \frac{t}{90}\right) = -180\left(1 - \frac{t}{90}\right)$

$= 2t - 180$ L/m

@ t=30 min

$\frac{dV}{dt} = 2 \times 30 - 180$

$= -120$ L/m

(d) $\frac{d^2V}{dt^2} = 2$ $\therefore \frac{dV}{dt}$ does not have any stationary points.

$\therefore$ Maximum rate must at be at either endpoint (as there are no points of inflexion)
$t=0$ $t=90$.

when $t = 0$ $\frac{dV}{dt} = -180$ L/m when $t = 90$ $\frac{dV}{dt} = 0$ L/m

$\therefore$ fastest draining rate when $t = 0$

P17

(Q36) $f(x) = x+1$ $g(x) = (x-1)^2$

Total Area = area under $f(x)\Big|_{-1}^{0}$ + area under $g(x)\Big|_{0}^{1}$

$$A = \int_{-1}^{0} (x+1)\,dx + \int_{0}^{1} (x-1)^2\,dx$$

$$= \left[\frac{x^2}{2} + x\right]_{-1}^{0} + \left[\frac{(x-1)^3}{3}\right]_0^1$$

$$= 0 - \left(\frac{(-1)^2}{2} + -1\right) + \left[0 - \frac{(-1)^3}{3}\right]$$

$$= \frac{1}{2} + \frac{1}{3}\ u^2$$

$$A = \frac{5}{6}\ u^2$$

(Q37) (a) $V = 2 - \frac{4}{t+1}$ when $t=1$ $V = 2 - \frac{4}{1+1}$

$= 2 - 2$

$= 0$

Velocity = 0 when $t = 1$
∴ at rest

(b) $d = \int v\,dt$

Q37 b) continued . .

$d = \int \left(2 - \frac{4}{t+1}\right) dt$

$= 2t - 4\ln(t+1) + C$ (when $t = 0$ $d = 1m$)

$\therefore 2 \times 0 - 4\ln(0+1) + C = 1$

$0 + C = 1$

$\therefore C = 1$

$\therefore d = 2t - 4\ln(t+1) + 1$

when at rest $v = 0$ and $t = 1$ (from part (a))

$\therefore d = 2 \times 1 - 4\ln(1+1) + 1$

$= 3 - 4\ln 2 = 0.227\,m$

(c) $a = \frac{dv}{dt} = \frac{d}{dt} 2 - \frac{4}{t+1}$

$= \frac{d}{dt} 2 - 4(t+1)^{-1}$

$= -1 \times -4\,(t+1)^{-2}$

$a = \frac{4}{(t+1)^2}$

Q37) d) The particle is ONLY at rest when $t=1$

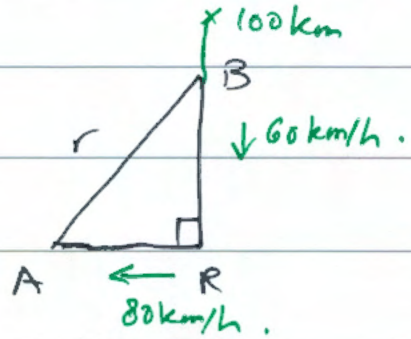
$\therefore$ The displacement curve only has 1 stationary point at $t=1$

$\therefore$ The acceleration at $t=1$ is $\frac{4}{(1+1)^2}$ which is positive.

$\therefore$ The displacement curve has a local minimum at $t=1$ (concave up) and the displacement $= 0.227$ which is positive $\therefore$

displacement always ≥ 0.227 $\therefore$ always positive.

Q38) a)



$RB = 100 - 60t$

$RA = 80t$

$r^2 = RB^2 + RA^2$

$= (100-60t)^2 + (80t)^2$

$= 10000 - 12000t + 3600t^2 + 6400t^2$

$= 10000t^2 - 12000t + 10000$

$r^2 = 2000(5t^2 - 6t + 5)$ as required

P20

(b) $\frac{d(r^2)}{dt} = \frac{d}{dt} 2000(5t^2-6t+5)$

$= 2000(10t-6)$

$= 20000t - 12000$ ✓ as required

(c) r a minimum when r^2 a minimum.

$\frac{d(r^2)}{dt} = 20000t - 12000 = 0$.

$\therefore \quad 2000(10t-6) = 0$

$t = \frac{6}{10} = 0.6$ hours

check to see if minimum.

$\frac{d^2(r^2)}{dt^2} = 2000(10) > 0 \quad \forall x$

$\therefore r^2$ curve concave up $\therefore$ stationary point local minimum

$\therefore r^2$ minimum when $t = 0.6$

$r = \sqrt{r^2}$

$= \sqrt{2000(5 \times 0.6^2 - 6 \times 0.6 + 5)}$

$= \sqrt{6400}$

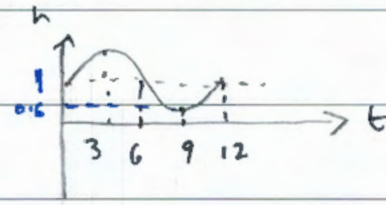
$r = 80\text{km}$ (minimum distance between cars).

Q39) (a) Period $= \frac{2\pi}{b}$ $\quad b = \frac{\pi}{6}$ $\quad h(t) = 1 + 0.4\sin\left(\frac{\pi}{6}t\right)$

$= 2\pi \div \frac{\pi}{6} = \frac{2\pi \times 6}{\pi} = \underline{\underline{12}}$

(b) minimum height $= 1 - 0.4$

$= \underline{0.6m}$

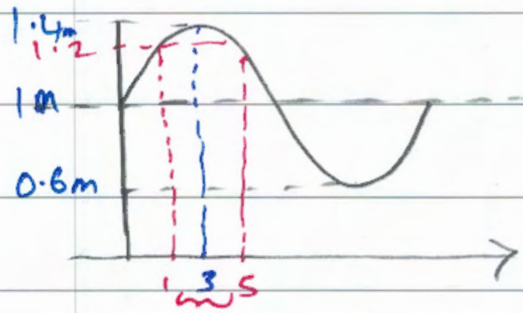


minimum occurs at $3/4$ of period $= 3/4 \times 12 = 9$ hours.

$\therefore$ time $= 8$ a.m $+ 9$ hours

$= \underline{5 \text{ P.m.}}$

(c) we want height $\geq 1.2\,m$



$1 + 0.4\sin\left(\frac{\pi}{6}t\right) \geq 1.2m.$

$0.4\sin(\pi/6\,t) \geq 0.2\,m$

$\sin(\pi/6\,t) \geq 1/2.$

$\pi/6 \leq (\pi/6\,t) \leq 5\pi/6$

$1 \leq t \leq 5$ $\quad \therefore$ ship can enter harbour between

$\underline{9 \text{ a.m. and } 1 \text{ p.m.}}$

$8 + 1 = 9$ a.m

$8 + 5 = 13$

$= 1$ p.m

THE END